

Solving Rational Equations

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Some students find solving rational equations difficult. Below is a short tutorial that aims to clarify the process.

The most common approach to solving rational equations is to convert them into the equality of two polynomials by developing the Least Common Denominator (LCD). By multiplying both parts of the equation by the LCD, the denominators are eliminated from all the terms of the equation, presenting them in a usual form that is easy to solve using conventional methods.

An important thing to keep in mind is that those **those variable values that cause the denomination to become 0, can not be solutions to the equation.** A number that looks to be a solution but causes the denominator of the original equation to become 0 is called an **extraneous solution.**

Example 1. Solve the following rational equation:

$$\frac{x}{x-1} - \frac{2}{x} = \frac{1}{x-1}$$

Caution: We can see right away that the values 1 and 0 could not be valid solutions, since if $x = 1$, or $x = 0$ then the denominator of the original equation has a value of 0!

Calculate and use the Lowest Common Denominator (LCD):

$$\frac{x^2}{x(x-1)} - \frac{2(x-1)}{x(x-1)} = \frac{x}{x(x-1)}$$

Multiply both sides of the equation by LCD to eliminate all of the denominators:

$$x^2 - 2x + 2 = x;$$

Collect like terms:

$$x^2 - 3x + 2 = 0;$$

Factor the equation as usual:

$$(x - 2)(x - 1) = 0;$$

The solutions appear to be:

$$x = 2; \quad x = 1;$$

As I mentioned above, the second solution is wrong, Therefore, the only solution to the equation is $x = 2$, with the extraneous solution being $x = 1$.

A similar problem with extraneous solutions might occur when solving proportions. A proportion is a special case of rational equation that expresses the equality of two ratios. In general, the statement

$$\frac{a}{b} = \frac{c}{d}$$

is a proportion. We use cross-multiplication to look for the solution:

$$ad = bc$$

Again, if the denominators of a proportion contain a variable, then such values of this variable that cause the denominator to become 0, can not be solutions to the equation.

Example 2. Solve the proportion.

$$\frac{1}{x - 2} = \frac{x}{10 - 5x}$$

Cross-multiply and solve the equation by factoring:

$$1(10 - 5x) = (x - 2)x;$$

$$10 - 5x = x^2 - 2x;$$

$$x^2 + 3x - 10 = 0;$$

$$(x + 5)(x - 2) = 0;$$

The solutions appears to be:

$$x = -5, \text{ and } x = 2;$$

Once again, the the only solution to the equation is $x = -5$, while $x = 2$ is an extraneous solution, because it causes the denominators of the original equation to become 0.